ARYAVART SHODH VIKAS PATRIKA
RNI TITLED NO. UPBBIL04292 PP. NO.- 466-471
RNI REG. NO. UPBIL/2014/66218
ISSN NO.-2347-2944 (Print)
e-ISSN NO.-2582-2454 (Online)
Vol.-13, No.-I, Issues-16, YEAR- June -2020

ANALYSIS OF A QUEUEING MODEL WITHROLLBACK/ RECOVERY STRATEGY

1. DR. RACHNA KHURANA, 2. DR. MANJU SHARMA

1. Assistant Professor, Department of Applied Sciences, Anand Engineering College, Agra, 2Associate Professor, Department of Mathematics, R. B. S. College, Agra (U.P.) India

Received - 21.05.2020, Revised - 25.05.2020, Accepted - 27.05.2020 Email :aaryavrat2013@gmail.com

Abstract: ***The queueing system considered in this paper consists two parallel load sharing processors. Two processors interact only during breakdown. When a processor fails, all the tasks in its queue are transferred to other processor that is in operating condition and new arrivals to the failed processor are rejected. All the jobs in the system are lost in the case both processors fail. The arrival process is assumed to be Poisson and service is of Coxian type with two stages. We have used an approximation method that produces highly accurate estimate of the throughput of the two processors. Using recursive method that utilizes generating function, we obtained steady state probabilities of the number of the tasks in each queue. This type of queues are quite common in big corporates such as international banks, communication companies and airlines.***

Key Words: Roll back, load sharing, breakdown, recursive methods, generating function.

Database management has evolved from a specialized computer application to a central component of a modern computing environment. A computer system, like any other mechanical or electrical device, is subject to failure. There is variety of causes of such failure, including disk crash, power failure, software error, a fire in the machine room or even sabotage. In each of these cases,information may be lost. Therefore, the data system manager must take actions in advance to ensure that the automicity and durability properties of transactions are preserved, inspite of such failures. An integral part of a data base system is a recovery scheme that is responsible for the restoration of the database to a consistent state that existed prior to the occurrence of failure. Checkpointing and Rollback procedures are used to preserve the data integrity. Checkpointing is the process of creating a persistent copy of the most recent state of a job or request while executed by a CPU. Rollback is a procedure to go back to the last persistent state of the current job when the CPU experiences failure. All the takes in it's queue are allocated to other processors that are in operating conditions.

In this paper we analyze two load sharing single server queues in a parallel processing environment. A failure in one queue transfers all the tasks to the other processor if it's server is not already down. Otherwise system is cleared of all the jobs. No new tasks arrive during the rollback recovery procedures. Using recursive method that utilizes generating function, we obtained steady state probabilities of the number of the tasks in each queue. This concept of load sharing also has application in manufacturing environment where machines are capable of processing different type of jobs.

There had been significant research related to failures and recovery process in the system. Altiok [1] studiesd queueing model of a single processor with failure.Altiok et. al. [2] developed a two node queueing model to approximate the throughput of each processor by uncorporating the task transfers among the mode. The arrival processes were Poisson and service processes were Erlang type. Chandy [3] did a significant survey of analytic models with Rollback and recovery strategy. Mikou et. al. [6] provide a comprehensive literature on interaction of two processes during failure.

Assumptions and Notations

For i=1 and 2

λ_i = interarrival rate of the customers exponentially distributed for node i.

μ_i = average service rate for node i, coxian distributedwhere $\frac{1}{\mu_i} = \frac{1}{\mu_{i_1}} + \frac{1-q}{\mu_{i_2}}$

Corresponding Author

ISSN NO.-2347-2944 (Print)
e-ISSN NO.-2582-2454 (Online)
Vol.-13, No.-I, Issues-16, YEAR- June -2020

μ_{i_1} = service rate at 1st stage

μ_{i_2} = service rate at 2nd stage

q = probability of bypassing second stage.

B_i = waiting line at node i.

δ_i = time rate until failure of node i.

γ_i = rate of repair activity starts immediately and lasts for an exponentailly distributed time.

$P_i(0)$ = steady state probability of processor being idle.

$P_i(b)$ = steady state probability of processor being busy.

$P_i(d)$ = steady state probability of processor being down, = $\frac{\delta_i}{\gamma_i+\delta_i}$, $P_i(d)+P_i(o)+P_i(b)=1$

$\bar{\sigma}_i$ = average throughput of node i= $\frac{P_i(b)}{E(x_i)}$

$N_i(t)$ = number of units in B_i

$P_2(n,j)$ = steady state probability that node 2 is in operating condition, n□0, j=1,2

δ_1' = rate of arriving units from node 1.

k = batch arrivals with size k with marginal probability $P_1(k)$ with rate δ_1'.

Calculation

In this study we developed a two node queueing model, who communicate each other when one experience failure. We decomposed the system into two separate node receiving batches of arrivals from the failure process of other. Here we are interested in obtaining the long-run average throughput $\bar{\sigma}_i = \frac{P_i(b)}{E(x_i)} = \frac{1-P_i(d)-P_i(0)}{E(x_i)}$. Thus we aim at obtaining an approximate value for $P_i(0)$ i=1,2.

For node 2 and phase 2 steady state flow balance equations are:

When n=0, j=1

$$\left[\lambda_2+\delta_1'+\delta_2\right]P_2(0) = q\,\mu_{2_1},P_2(1,1)+(1-q)\,\mu_{2_2}\,P_2(1,2) \qquad \dots(1)$$

When n□1, j=1

$$[\lambda_2+\mu_2+\delta_2+\delta_2']P_2(n,1) = \lambda_2 P_2(n-1,1)+\mu_{2_1},qP_2(n+1,1)$$
$$+\mu_{2_2}(1-q)P_2(n+1,2)+\delta_1'\sum_{k=1}^{n} C_1(k)P_2(n-k,1) \qquad \dots(2)$$

Similarly steady state equation for n□1, j=2

$$\left[\lambda_2+\mu_2+\delta_2+\delta_1^{'}\right]P_2(n,2)=\lambda_2\,P_2(n-1,2),+\mu_{2_1}(1-q)P_2(n,1)$$
$$+\delta_1^{'}\sum_{k=1}^{n-1}C_1(k)P_2(n-k,2) \qquad \ldots(3)$$

Here

$P_2(0,1)= P_2(0)$

& $P_2(0,i)=0$ for i >1

We define the probability that k units are transferred from queue 1 to queue 2 as $C_1(k)$.

Here $C_1(k)$ can be approximated as

$$\frac{P_1(k)}{1-P_1(0)-P_1(d)}\text{fork=1,2} \qquad \ldots(4)$$

The above steady state flow balance equations can be solved for $P_2(n,j)$ recursively

Substituting n=1 in equation (3)

$$\left[\lambda_2+\mu_2+\delta_2+\delta_1^{'}\right]P_2(1,2)=(1-q)\mu_{2_1}\,P_2(1,1) \qquad \ldots(5)$$

From equation (1)

$$\left[\lambda_2+\delta_2+\delta_1^{'}\right]P_2(0)=q\,\mu_{2_1},P_2(1,1)+(1-q)\mu_{2_2},P_2(1,2)+\gamma_2P_2(d)$$

$$\Rightarrow P_2(1,2)=\frac{1}{(1-q)\mu_{2_2}}\left[\left(\lambda_2+\delta_2+\delta_1^{'}\right)P_2(0)-q\,\mu_{2_1}P_2(1,1)-\gamma_2P_2(d)\right] \qquad \ldots(6)$$

Using equation (6) in (5) $P_2(1,1)$ can be defined as

$$P_2(1,1)=\frac{\left(\lambda_2+\mu_2+\delta_2+\delta_1^{'}\right)}{(1-q)^2\,\mu_{2_1}\cdot\mu_{2_2}}\left[\left(\lambda_2+\delta_2+\delta_1^{'}\right)P_2(0)-q\,\mu_{2_1}P_2(1,1)-\gamma_2P_2(d)\right] \qquad \ldots(7)$$

From equation (7) we conclude $P_2(n,j)$ can be defined as recurrive expression

$$\tilde{P}_2(n)=AR_n, \qquad n=1,2..... \qquad \ldots(8)$$

Where $\tilde{P}_2(n)$ is a vector of steady state probabilities of having n units in queue 2 over all processing time phases, i.e.

$$\tilde{P}_2(n)=\begin{pmatrix}P_2 & (n,1)\\ P_2 & (n,2)\end{pmatrix}$$

When n=1

$$P_2(1)=\begin{pmatrix}P_2 & (1,1)\\ P_2 & (1,2)\end{pmatrix}=A.R_1$$

Where

$$P_2(2)=\begin{pmatrix}P_2 & (2,1)\\ P_2 & (2,2)\end{pmatrix}=A.R_2$$

ISSN NO.-2347-2944 (Print)
e-ISSN NO.-2582-2454 (Online)
Vol.-13, No.-I, Issues-16, YEAR- June -2020

A is as same as defined in equation (9) and R_2 is defined as

$$R_2 = \begin{bmatrix} \lambda_2 P_2(1,2) - \delta_1^{'} C_1(1) P_2(1,2) \\ (\lambda_2 + \mu_2 + \delta_2 + \delta_1^{'}) P_2(1,1) - \lambda_2 P_2(0) \\ -\delta_1^{'} C_1(1) P_2(0) - \mu_{2_1} q P_2(2,1) \end{bmatrix} \quad \ldots(11)$$

In general for n>1

when i=1

$$R_n(1) = -\lambda_2 P_2(n-1,2) - \delta_1^{'} \sum_{k=1}^{n-1} C_1(k) P_2(n-k,2) \quad \ldots(12)$$

when i=2

$$R_n(2) = (\lambda_2 + \mu_2 + \delta_2 + \delta_1^{'}) P_2(n-1,1) - \lambda_2 P_2(n-2,1)$$

$$-\mu_{2_1} q_{n-1} P_2(n,1) - \delta_1^{'} \sum_{k=1}^{n-1} C_1(k) P_2(n-k-1,1) \quad \ldots.(13)$$

So now using the above equations we can obtain all $\overline{P}_2(n), n = 1,2$ ...However $P_2(0)$ is still required to evaluate. For this we use partial generating function and its properties. Let us assume $G_2(z,2)$ denotes partial generating function of $P_2(n,2)$ at Coxian stage 2, and $C_1(z)$ denote generating function of no. of units transferred from node i i.e.

$$G_2(z,2) = \sum_{n=1}^{\infty} Z^n P_2(n,2) \; and \; C_i(z) = \sum_{n=1}^{\infty} Z^n C_i(n)$$

Let $G_2(z)$ denotes gererating function of distribution of the number in the system.

$$G_2(z) = P_2(0) + \sum_{n=1}^{\infty} G_2(z,k)$$

Or $G_2(z) = P_2(0) + G_z(z,1) + G_2(z,2)$

With $G_2(1) = 1 - P_2(d)$

Now with the help of equation (2)

$$G_2(z,1) = \frac{\left[\lambda_2(z) + \delta_1^{'} C_1(z)\right] P_2(0) - \mu_{2_2}(1-q) P_2(1,2) - \mu_{2_1} q P_2(1,1) + \frac{\mu_{2_2}(1-q)}{z} G_2(z,2)}{(\lambda_2 + \mu_2 + \delta_2 + \delta_1^{'}) - \lambda_2 z - \delta_1^{'} C_1(z) - \frac{\mu_{2_1} q}{z}} \quad \ldots(14)$$

Using equation (3) for obtaining $G_2(z,2)$

$$G_2(z,2) = \frac{\mu_{2_1}(1-q)}{(\lambda_2 + \mu_2 + \delta_2 + \delta_1^{'}) - \lambda_2 z - \delta_1^{'} C_1(z)} G_2(z,1) \quad \ldots(15)$$

with the help of equation (1), (14) and (15)

ISSN NO.-2347-2944 (Print)
e-ISSN NO.-2582-2454 (Online)
Vol.-13, No.-I, Issues-16, YEAR- June -2020

$$G_2(z,1)=\frac{z\left[(1-z)\lambda_2+\{1-C_1(z)\}\delta_1^{'}+\delta_2+\mu_2\right]\left[\{\lambda_2(z-1)+\delta_1^{'}\{C_1(z)-1\}-\delta_2\}P_2(0)+P_2(1,1)\mu_{2_1}q+\gamma_2P_2(d)\right]}{\left[z\{\mu_2+(1-z)\lambda_2+\{1-C_1(z)\}\delta_1^{'}+\delta_2\}^2-\mu_{2_1}q\{(1-z)\lambda_2+\{1-C_1(z)\}\delta_1^{'}+\delta_2+\mu_2\right]}+\mu_{2_1}\mu_{2_2}(1-q)^2$$

So finally $G_2(z)$ can be evaluated easily using

$G_2(z)=P_2(0)+G_2(z,1)+G_2(z,2)$

$$G_2(z)=\left[\mu_2^2+2\mu_2\{(1-z)\lambda_2+(1-C_1(z))\delta_1^{'}+\delta_2\}-\frac{\mu_{2_1}q}{z}\{(1-z)\lambda_2+(1-C_1(z))\delta_1^{'}+\delta_2+\mu_2\}-\frac{\mu_{2_1}\mu_{2_2}(1-q)^2}{z}\right]P_2(0)$$

$$-\mu_2\left[\lambda_2(1-z)+(1-C_1(z))\delta_1^{'}+\delta_2\right]P_2(0)+$$

$$\frac{\left[(1-z)\lambda_2+(1-C_1(z))\delta_1^{'}+\delta_2+\mu_2\right]\left[P_2(1,1)\mu_{2_1}q+\gamma_2P_2(d)\right]}{\left[\{(1-z)\lambda_2+(1-C_1(z))\delta_1^{'}+\delta_2+\mu_2\}^2-\frac{\mu_{2_1}q}{z}\{(1-z)\lambda_2+(1-C_1(z))\delta_1^{'}+\delta_2+\mu_2\}\frac{\mu_{2_1}\mu_{2_2}(1-q)^2}{z}\right]}$$

Since the coefficient of $P_2(0)$ is 0 at$z$=0, the numerator of $G_2(z)$ has exactly the same number of zeros as its denominator and hence has same roots. Afterward $P_i(0)$ can be obtained at the zeros of numerotor from the denominator of $G_2(z)$.

Denominator approximately can be written as:

$$f_2(z)=\{\mu_2+\delta_2+\lambda_2(1-z)+(1-C_1(z))\delta_1^{'}\}-\frac{\mu_2}{z}=[g(z)+h(z)]-\mu_2/z$$

where

$$g(z)=\mu_2\left(1-\frac{1}{z}\right)+\lambda_2(1-z)+\delta_2$$

$$h(z)=\delta_1^{'}(1-C_1(z))+\mu_2/z$$

Now since $g(z)$ has only one root in [0,1] so f_2 will also have one root in [0,1] according to Rouche's theorem say the root be $z=z'$.

Thus $P_2(0)$ can be evaluated from the numerator of $G_2(z)$ at $z=z'$.

CONCLUSION

We have considered a queueing model which consists of two parallel servers who share the laod of customers in event of breakdown of a server. If both the servers suffer breakdown, all the customers are lost. Approximation method is used to find the long-run average throughput of the processors.

ISSN NO.-2347-2944 (Print)
e-ISSN NO.-2582-2454 (Online)
Vol.-13, No.-I, Issues-16, YEAR- June -2020

REFRENCES

1. Alitiok, T.(1989): "Queueing modeling of a single processor with failure". Performance Evaluation, 9, 93-102.
2. Altiok, T., Duy, C.V. and Baykal, G.M. (1998): "Two load sharing processes with failures". Computers Ops. Res., 25, 183-189.
3. Chandy, K.M.(1975): "A survey of analysis models of Rollback and recovery strategy". Computers, 5, 40-47.
4. Gross, D. & Harris, C.M., Fundamentals of Queueing theory. John Wiley and Sons, New York, 1974.
5. Kleinrock, L., Queueing System, Vol. I and II, John Wiley and Sons, New York, 1974.
6. Mikou, N., Idrissi-Kacimi, O. and Saadi, S. (1995): "Two Processes interacting only during beak down". Queueing System, 19, 301-317.
7. Rudin, W., Real and Complex Analysis, 2nd ed. McGraw Hill, 1974.

*****

PIF/5.002 *ASVS Reg. No. AZM 561/2013-14*