ARYAVART SHODH VIKAS PATRIKA
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# HYDROMAGNETIC FREE CONVECTIVE FLOW OF STRATIFIED VISCOUS FLUID PAST A POROUS VERTICAL PLATE

**KUMAR, V.**

Deptt. of Mathematics, K.A. (PG) College, Kasganj

***Abstract : Unsteady hydromagnetic free convection of, incompressible, electrically conducting, viscous, stratified liquid in a porous medium past a vertical, porous, isothermal infinite plate fluctuating with time dependent suction velocity is presented. The flow is considered under the influence of transversely applied uniform magnetic field. It is assumed that the magnetic Reynolds number is very small so that the induced magnetic field is negligible. To obtain the solution for the velocity field and temperature field of the problem. The technique suggested by Lighthill (1954) is used. In addition skin- friction coefficient at the plate and heat transfer in terms of Nusselt number are as obtained. The results obtained are discussed.***

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**1. Introduction :** The solution of fluid flow with different flow conditions and the corresponding energy equation with different wall temperature including heat generated due to friction etc. is well reported in literature. At present fluid flow through porous media is of considerable importance in a wide range of disciplines in science and technology viz., in soil mechanise, ground water hydrology petroleum engineering, water purification, industrial filtration, ceramic engineering and powder metallurgy. The importance has been recongnised in agricultural engineering to study the under ground water resources, seepage of water in river beds, in chemical engineering for filtration and purification process in petroleum technology to study the movements of natural gas, oil and water through the oil reservoirs. The theory of laminar flow through homogeneous porous media is based on an experiments originally conducted by Darcy, Ahmadi & Manvi (1971) gave a general equation governing the motion of a viscous fluid through rigid porous medium and applied the results obtained to some basic flow problems.

The flow problems on convective flow have been studied by several authors Gersten & Gross (1974), Yamamoto & Iwamura (1976), Ram & Mishra (1977), Raptis & Perdikis (1985), Singh & Singh (1989), Purushothaman et.al (1990), Singh & Rana (1992) and Singh et.al (1993, 97, 99, 2000, 2001) have studied convective flow problems under different physical situations and corresponding boundary conditions.

The study of flow through porous media with stratified flow is of consideration interest due to its application in soil sciences, petroleum industry, chemical engineering, filtration processes and so on. The study of stratified fluid flows presence of uniform magnetic field has its application in astrophysics, separation oils of different densities, and several other application in engineering and technology. The study of effects of various physical variables on stratified viscous flow has been studied by so many authors including Singh & Singh (1989, 91) Ram et.al (1993), Das & Nandy (1995), soundalgekar and uplekar (1995)soundalgekar et.al (1996), Singh et.al (1998) and several other authors.

Unsteady hydromagnetic free convection of an incompressible, electrically conducting, viscous, stratified liquid in a porous medium past a vertical, porous, isothermal infinite plate fluctuating with time dependent suction velocity is presented. The flow is considered under the influrance of transversely applied uniform magnetic field. It is assumed that the magnetic Reynold number is very small so that the induced magnetic field is negligible. To obtain the solution for the velocity field and temperature field of the problem, the technique suggested by Lighthill (1954) is used. In addition skin-friction coefficient at the plate and heat transfer in terms of Nusselt number are also obtained. The results obtained are discussed.

**Corresponding Author**

**2. Formulation of the Problem :**

We consider the unsteady convective flow of an incompressible, electrical conducting, viscous, stratified fluid through a porous medium past an vertical porous, isothermal infinite plate with time dependent suction at the plate under magnetic applied normal to the flow. Let *x*-axis be in the plane of the plate along the direction of the flow and *y*-axis perpendicular to the plate and passes through the *x*-axis. We further assume that the density ($\rho$), viscosity ($\mu$), thermal conductivity ($K_T$) and volumetric coefficient of thermal expansion ($\beta$) satisfy exponential law viz. $\rho = \rho_0 \exp(-\beta y)$, $\mu = \mu_0 \exp(-\beta y)$, $K_T = K_{T_0} \exp(-\beta y)$ and $\beta' = \beta'_0 \exp(-\beta y)$ where $\beta$ is the stratification factor and *y* is the distance perpendicular to the plate. In addition, the magnetic field $B = B_0 \exp\left(-\frac{\beta y}{2}\right)$ is applied perpendicular to the flow region and the suction velocity $v = v_0\left[1 + \in f(t)\right]$ is a function of time and the pressure gradient is negligible. In addition to above considerations, the present analysis is made on the basis of following assumptions :

(i) All the fluid properties are constant except the influence of the density variation with temperature only in body force term.

(ii) In the influence of density variation, other terms of the momentum equation and variation of expansion coefficient with temperature are considered negligible.

(iii) The free convection currents are in existence due to temperature difference ($T - T_\infty$).

Hence, the governing equations of motions and energy for the flow of fluids under the present configuration are :

Momentum Equation :

$$\rho\left[\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y}\right] = \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) - \frac{\mu}{k}u - \sigma B^2 u + g\beta'\left(T - T_\infty\right) \qquad ....(2.1)$$

Energy Equation :

$$\rho C_P\left[\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial y}\right] = K_T\frac{\partial^2 T}{\partial y^2} \qquad ....(2.2)$$

where- $u$ = the velocity in the direction of *x*-axis.
$T$ = the temperature of the liquid.
$T_\infty$ = the temperature of the fluid far from the plate.
$\beta'$ = the volumetric coefficient of thermal expansion.
$\sigma$ = the conductivity of the liquid.
$K_T$ = the thermal conductivity.
$C_p$ = the specific heat at constant pressure.

Introducing the values of $\rho$, $\mu$, $K_T$, $\beta'$ and $\overline{B}$ we obtain.

$$\frac{\partial u}{\partial t} - v_0\left[1 + \in f(t)\right]\frac{\partial u}{\partial y} = \upsilon_0\frac{\partial^2 u}{\partial y^2} - \upsilon_0\beta\frac{\partial u}{\partial y} - \frac{\upsilon_0}{K}u - \frac{\sigma B_0^2}{\rho_0}u + g\beta_0\left(T - T_\infty\right) \quad ....(2.3)$$

$$\frac{\partial u}{\partial t} - v_0\left[1 + \in f(t)\right]\frac{\partial T}{\partial y} = \frac{K_{T_0}}{C_p}\frac{\partial^2 T}{\partial y^2} \quad ....(2.4)$$

The boundary condition for the present problem are

$$u = v_0\left[1+\in f(t)\right], \quad T = T_w\left[1+\in f(t)\right], \quad at \quad y=0$$
$$....(2.5)$$
$$u\to 0, \quad T\to T_\infty, \quad as \quad y\to\infty$$



We introduce the following non-dimensional quantities :

$$u^* = \frac{u}{v_0}, \quad y^* = \frac{y v_0}{\upsilon_0}, \quad t^* = \frac{t v_0^{\,2}}{\upsilon_0^{\,2}}, \quad k^* = \frac{k v_0^{\,2}}{\upsilon_0^{\,2}} \quad and \quad T^* = \frac{T - T_\infty}{T_w - T_\infty}$$

Introducing the above non-dimensional variables, the equations (2.3) and (2.4) reduce to

$$\frac{\partial u}{\partial t} - \left[1+\in f(t)\right]\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} - S\frac{\partial u}{\partial y} - \left(M^2 + \frac{1}{k}\right)u + G_r T \quad ....(2.6)$$

$$\frac{\partial T}{\partial t} - \left[1 + \in f(t)\right]\frac{\partial T}{\partial y} = \frac{1}{P_r}\frac{\partial^2 T}{\partial y^2} \quad ....(2.7)$$

where $G_r = \dfrac{g\beta_0\left(T_w - T_\infty\right)\upsilon_0}{v_0^{\,3}}$ (Grashof number)

$S = \dfrac{\beta\upsilon_0}{v_0}$ (Stratification factor)

$M = \dfrac{\sigma B_0}{v_0}\sqrt{\dfrac{\sigma\upsilon}{\rho_0}}$ (Magnetic parameter)

and $P_r = \dfrac{\mu_0 C_p}{K_{T_0}}$ (Prandtl number)

**3. <u>Solution of the Problem</u> :**

To obtain the solution of the problem, we consider the case when the suction velocity is exponentially decreasing function of time. Hence we assume

$$f(t) = e^{-nt} \quad ....(3.1)$$

Substituting (3.1) in the equations (2.6) and (2.7), we get

$$\frac{\partial u}{\partial t} - \left[1 + \in e^{-nt}\right]\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} - S\frac{\partial u}{\partial y} - \left(M^2 + \frac{1}{8}\right)u + G_r T \quad ....(3.2)$$

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$$\frac{\partial T}{\partial t} - \left[1 + \in e^{-nt}\right]\frac{\partial T}{\partial y} = \frac{1}{P_r}\frac{\partial^2 T}{\partial y^2} \quad ....(3.3)$$

The boundary conditions (2.8) reduce to

$$u = 1 + \in e^{-nt}, \quad T = 1 + \in e^{-nt}, \quad at \quad y = 0$$

$$u \to 0, \quad T \to 0, \quad as \quad y \to \infty \quad ....(3.4)$$

Following Lighthill (1954), we assume the velocity field and the temperature field as :

$$u(y,t) = u_0(y) + \in u_1(y) e^{-nt}$$

$$T(y,t) = T_0(y) + \in T_1(y) e^{-nt} \quad ....(3.5)$$

Substituting (3.5) in the equations (3.2) and (3.3) and equating the harmony and non-harmonic terms, we get

$$u_0^{''}(g) + (1 - S)u_0^{'}(y) - M_1 u_0(y) = -G_r T_0 \quad ....(3.6)$$

$$u_1^{''}(y) + (1 - S)u_1^{'}(y) - (M_1 - n)u_1(y) = -G_r T_1 - u_1^{'} \quad ....(3.7)$$

$$T_0^{''}(y) + P_r T_0^{'}(y) = 0 \quad ....(3.8)$$

$$T_1^{''}(y) + P_r T_1^{'}(y) + nP_r T_1(y) = -P_r T_0^{''} \quad ....(3.9)$$

The boundary conditions (3.4) are transformed to :

$$u_0 = 1, \quad u_1 = 1, \quad T_0 = 1, \quad T_1 = 1, \quad at \quad y = 0$$

$$u_0 \to 0, \quad u_1 \to 0, \quad T_0 \to 0, \quad T_1 \to 0 \quad as \quad y \to \infty \quad ...(3.10)$$

The solution of the above coupled equations, using the corresponding boundary conditions is obtained as

$$T_0(y) = e^{-P_r y} \quad ....(3.11)$$

$$T_1(y) = \left(1 - \frac{P_r}{n}\right)e^{-M_2 y} + \frac{P_r}{n}e^{-P_r y} \quad ....(3.12)$$

$$u_0(y) = (1 + K_1)e^{-M_2 y} - K_1 e^{-P_r y} \quad ....(3.13)$$

$$u_1(y) = (1 + K_3 + K_5 + K_6)e^{-M_4 y} - K_3 e^{-M_2 y} - K_5 e^{-M_3 y} - K_6 e^{-P_r y} \quad ....(3.14)$$

where $M_1 = M^2 + \frac{1}{k}$, $\quad M_2 = \frac{P_r + \sqrt{P_r(P_r + 4n)}}{2}$

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$$M_4 = \frac{(1-S) + \sqrt{(1-S)^2 + 4(M_1 - n)}}{2}$$

$$K_1 = \frac{G_r}{P_r^2 - (1-S)P_r - M_1},$$

$$K_3 = \frac{G_r\left(1 - \frac{P_r}{n}\right)}{M_2^{\,2} - (1-S)M_2 - (M_1 - n)}$$

$$K_3 = \frac{(1-K_1)M_3}{M_3^{\,2} - (1-S)M_3 - (M_1 - n)}$$

$$K_6 = \frac{\left(\frac{G_r P_r}{n} + K_1 P_r\right)}{P_r^2 - (1-S)P_r - (M_1 - n)}$$

Substituting the values of $u_0(y),\ u_1(y),\ T_0(y)$ and $T_1(y)$ from (3.13), (3.14), (3.11) and (3.12) in (3.1), we obtain :

$$u(y,t) = (1+K_1)e^{-M_2 y} - K_1 e^{-P_r y} + \in\Big[(1+K_3+K_5+K_6)e^{-M_4 y} - K_3 e^{-M_2 y} - K_5 e^{-M_3 y} - K_6 e^{-P_r y}\Big]e^{-nt} \qquad ....(3.15)$$

$$T(y,t) = e^{-P_r y} + \in\left[\left(1 - \frac{P_r}{n}\right)e^{-M_2 y} + \frac{P_r}{n}e^{-P_r y}\right]e^{-nt} \qquad ....(3.16)$$

**4. <u>Skin-Friction and Heat Transfer</u> :**The skin fricition coefficient () at the plate at $y = 0$ is :

$$\tau = \left(\frac{\partial u}{\partial y}\right)_{y=0} = K_8 + \in K_{9e}^{-nt} \qquad .....(4.1)$$

The rate of heat transfer in terms of Nusselt number ($N_u$) at the vertical plate at $y = 0$ is :

$$N_u = -\left(\frac{\partial T}{\partial y}\right)_{y=0} = P_r + K_{10} \in e^{-nt} \qquad ....(4.2)$$

Where $K_8 = P_r K_1 - (1 + K_1)M_2$

$$K_9 = K_3 M_2 + K_5 M_3 + K_6 P_r - (1 + K_3 + K_5 + K_6)M_2$$

and $$K_{10} = \left(1 - \frac{P_r}{n}\right)M_2 + \frac{P_r^2}{n}$$

**TABLE – 1**
Effects of $P_r$, $M$, $S$ and $G_r$ on skin-friction
($k_0$ = 5.0, $i$ = 1.0, $n$ = 2.0 and $\in$ = 0.002)

| $P_r$ | $M$ | $S$ | $G_r$ | $\tau$ |
|---|---|---|---|---|
| 0.71 | 2.0 | 1.0 | 5.0 | – 0.10386 |
| 7.00 | 2.0 | 1.0 | 5.0 | – 8.77937 |
| 0.71 | 3.0 | 1.0 | 5.0 | – 1.06540 |
| 0.71 | 2.0 | 2.0 | 5.0 | 0.35602 |
| 0.71 | 2.0 | 1.0 | 12.0 | 1.99359 |

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**TABLE – 2**

Effects of $P_r$, $n$ and $t$ on rate of heat transfer

| $P_r$ | $N$ | $t$ | $N_u$ |
|---|---|---|---|
| 0.71 | 2.0 | 1.0 | 0.71872 |
| 7.00 | 2.0 | 1.0 | 7.14182 |
| 0.71 | 3.0 | 1.0 | 0.71797 |
| 0.71 | 2.0 | 2.0 | 0.73532 |

**5. Discussion and Consulations :** To observe the physical depth of the problem, the effects of Prandtl number ($P_r$), magnetic parameter ($M$), stratification factor ($S$) and Granshof number ($G_r$) on steady part of velocity are noted at $k_0 = 5.0$, $t = 1.0$, $n = 2.0$ $\in = 0.002$ and are depicted in Fig. 1 and the effects of Prandtl number ($P_r$), magnetic parameter ($M$), stratification factor ($S$) and Grashof number ($G_r$) on time dependent part of velocity $k_0 = 5.0$, $t = 1.0$, $n = 2.0$ and $\in = 0.002$ are depicted in Fig. 2. The effect of Prandtl number ($P_r$) on steady part of temperature ($T_0$) is shown in Fig. 3 and of time dependent part of temperature ($T_1$) is shown in Fig. 4. The effect of the said parameters on skin-friction ($\tau$) are represented in Table–1 and the effects of Prandtl number ($P_r$), frequency parameter ($n$) and time parameter ($t$) on rate of heat transfer ($N_u$) are represented in Table–2. The conclusions of the study are a follows :

1. An increase in $S$ or $G_r$ increases the steady part of velocity while an increase $P_r$ or $M$ decreases the steady part of velocity.
2. An increase in $G_r$ increases the time dependent part of velocity while at increase $P_r$, $S$ or $M$ decreases the time dependent part of velocity.
3. An increase in the value of Prandtl number decreases the steady part of temperature.
4. An increase in the value of Prandtl number decreases the time dependent part of temperature.
5. An increase in $P_r$ or $M$ decreases the skin-friction at the plate while an increase $S$ or $G_r$ increases the skin-friction at the plate.
6. An increase in $P_r$ increases the rate of heat transfer while an increase in $n$ or $t$ decreases the rate of heat transfer.

Figure 3. Effect of Prandtl Number on $T_0$

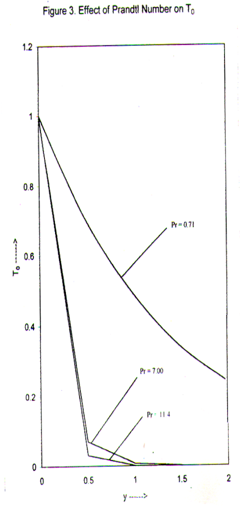



Figure 4. Effects of Prandtl Number on $T_1$

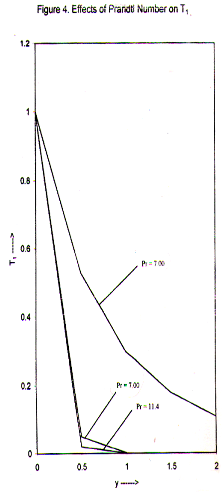



Figure 1. Effects of Pr, M, S and Gr on Steady Part of Velocity

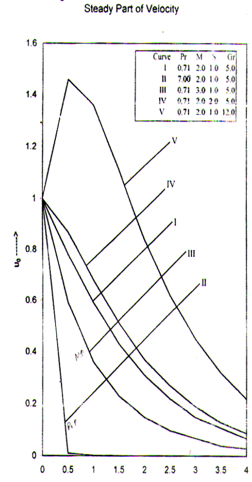
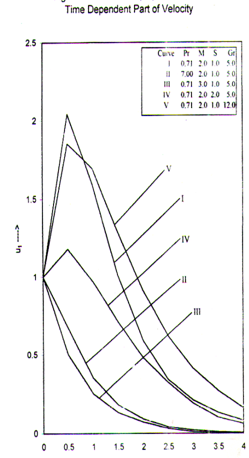



Figure 2. Effects of Pr, M, S and Gr on Time Dependent Part of Velocity

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