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A MULTI-CLASS BATCH ARRIVAL RETRIAL QUEUE WITH SERVICE IN BULK

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Abstract:

This paper deals with the repeated order $M^{X_i}/G^B/1$ queueing system in which customers belonging to n different classes arrive in batches of random size at a single channel retrial queueing system. Service is provided in batches of maximum size B of the same class. If an arriving batch finds the channel free, then maximum B number of customers are taken for service and remaining form the source of repeated order customers. While on arrival of a batch, if the server is busy then all of the customers of that batch join the source of repeated order customers. Closed form expressions are obtained for the expected queue lengths of various customer classes in steady state.This type of queueing model finds its application in computer communication networks and tele-communication networks.

Key Words: Retrial queue, Batch arrival, Batch service, Multi-class, generating function.

It is a common experience while trying to contact someone either by mobile or telephone that repeated attempts are made by usif we are not connected in a single trial. It also happens when we try to reach a link on internet for getting some information. If we do not succeed in a single trial, then we try again and again to reach the required website. These are examples of a retrial queue in which customers are allowed to retry till it get served.Although these are the typical examples we encounter in computer and tele-communication network, but we may find similar situation in many more queuing networks like production and transportation. Retrial queues are studied by many authors.Falin(1988) worked on a multiclass batch arrival retrial queue. Aissani (2000) studied M^x /G/1 retrial queue.Alpha &Isotupa (2004) considered an M/PH/k retrial queue with finite number of sources. Amador & Artalejo (2009)studied customer behavior in an M/G/1 retrial queue.Kim et.al.(2010) took tandem retrial queuing model.Choudhary &Ke(2012) have studied a batch arrival retrial queue. Chen et.al.(2016) also considered batch arrival retrial queue , but with non-persistent customers. Ayyappan &Thamizhselvi (2017) analyzed bulk arrival retrial queue with priority, breakdown and repair. In this paper an attempt has been made to study a multiclass batch arrival and batch service retrial queue.

This paperconsiders a queuing system with a single server at which customers belonging to n different classes arrive. The arrival times of demands of the ith type (i-demands) form a Poisson process with rate λ_i , at every arrival epoch with certain probability C_{ik} exactly k i-demands arrive. We call these demands as primary calls. If an arriving batch of i-customers of size k finds the channel free and k □ B then all of the customers will be taken into service immediately. If an arriving batch is of size k > B,then B out of the k customers are selected randomly from a batch and will occupy the channel and rest of the customers i. e. k-B of that batch will form the sources of repeated i-calls (i-sources). Customers from every such source produces a Poisson process of repeated calls with intensity μ_i>0. So, if an incoming group of customers finds a free line it is served as stated above and those who are served leave the system after service, whereas remaining form the source of repeated i-calls. Otherwise, if the channel is engaged, the system state does not change. Service times, for primary and for repeated i-calls, have the same distribution function B_i (X). Interarrival period, batch sizes, retrial times and service times are supposed to be mutually independent.

NOTATIONS

Let us define

$b_i(x) = B_i^1(X) / \{1- B_i(X)\}$ be the instantaneous service intensity of i-calls.

□ (s) = $\int_0^\infty exp(-sx)dB_i(X)$ be the Laplace-Stieltjes transform of the service time distribution function B_i (X).

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$\beta_{ik} = (-1)^k \beta_i^{(k)}(0)$ be the kth initial moment of the i-calls service time.

$C_i(Z) = \sum_{k=1}^{\infty} C_{ik} Z^k$, $\overline{C_i}$, σ_i^2 be respectively the generating function, the mean and the variance of batch size of i-calls.

$\rho_i = (\lambda_i \overline{C_i}\, \beta_{i1})/B$ be the system load due to primary i-calls and □= $\sum_{i=1}^{n} \rho_i$.

□= □$_1$ + □$_2$ +…+ □$_n$.

□(s) = $\int_0^{\infty} exp(-sx)\ dB(X)$, □$_k$ = $(-1)^k$ □$^{(k)}(0)$.

C(t) = 0 states that channel is free at the time t.

C(t) = i states that channel is occupied by some i-calls at time t.

$N_i(t)$ denotes the number of i-sources at time t.

If C(t) ≠ 0, then ∅ (t) is the time during which the channel has been serving the calls which occupies the channel at time t.

We shall consider the system in steady -state, which exists if and only if □< 1, so the condition □< 1 is assumed to be hold . Our aim is to find the mean queue lengths, $N_i = E[N_i(t)]$, 1□i □n.

Let $m = (m_1, m_2, \ldots, m_n)$, $Z = (Z_1, Z_2, \ldots, Z_n)$ be the n-dimensional vectors.

$e_i = (0,\ldots,1,\ldots,0)$ be the n-dimensional vector with ith coordinate equal to one and the rest equal to zero, and $e = (1,1,\ldots,1)$ be the n- dimensional vector which has all coordinates equal to one.

Considering the system in steady-state, we define

$P_0(m) = P\{C(t) = 0, N_1(t) = m_1, \ldots, N_n(t) = m_n\}$

$P_i(m, x)\,dx = P\{C(t) = I,\ x < \emptyset(t) < x+dx,\ N_1(t) = m_1, \ldots, N_n(t) = m_n\}$ i=1,2,…,n.

MEAN QUEUE LENGTHS

The equations of statistical equilibrium are obtained as follows:

$$[\lambda + \sum_{i=1}^{n} \mu_i\, m_i]\, P_0(m) = \sum_{i=1}^{n} \int_0^{\infty} P_i(m,x)\, b_i(x)\, dx \quad (1)$$

$$\frac{dP_j(m,x)}{dx} = -[\square + b_j(x)]\, P_j(m,x) + \sum_{i=1}^{n} \lambda_i \sum_{k=1}^{m_i} C_{ik}\, P_j(m - ke_i, x) \quad (2)$$

$$P_j(m,0) = \lambda_j \sum_{k=B}^{B+m_j} C_{ik}\, P_0(m - (k-B)e_j) + \sum_{k=1}^{B} \mu_j (m_j + k)\, P_0(m + k\, e_j), m_j \geq 1 \quad (3)$$

$$P_j(m,0) = \lambda_j \sum_{k=1}^{B} C_{jk}\, P_0(m) + \sum_{k=1}^{B} \mu_j\ k\, P_0(m + k\, e_j), \quad m_j = 0 \quad (4)$$

By defining the generating functions

$$\emptyset_0(Z) = \sum_{m_1=0}^{\infty} \cdots \sum_{m_n=0}^{\infty} Z_1^{m_1} \ldots Z_n^{m_n} P_0(m)$$

$$\emptyset_i(Z, x) = \sum_{m_1=0}^{\infty} \cdots \sum_{m_n=0}^{\infty} Z_1^{m_1} \ldots Z_n^{m_n} P_i(m,x)$$

we get from equations (1), (2), (3) and (4)

$$\lambda\emptyset_0(Z) + \sum_{i=1}^{n}\mu_i Z_i \frac{\partial\emptyset_0(Z)}{\partial Z_i} = \sum_{i=1}^{n}\int_0^\infty \emptyset_i(Z,x)\, b_i(x)\, dx \quad (5)$$

$$\frac{\partial\emptyset_j(Z,x)}{\partial x} = -[\sum_{i=1}^{n}\lambda_i\,(1 - C_i(Z_i)) + b_j(x)]\,\emptyset_j(Z,x) \quad (6)$$

$$\emptyset_j(Z,0) = \frac{\lambda_j}{Z_j{}^B}\quad [C_j(Z_j) - \sum_{k=1}^{B-1} Z_j{}^k C_{jk}]\emptyset_0(Z) + \mu_j \sum_{k=0}^{B-1}\frac{1}{Z_j{}^k}\frac{\partial\emptyset_0(Z)}{\partial Z_j} - \prod A(Z_j)$$

(7)

where □ and $A(Z_j)$ are defined as follows:

$$□ = \sum_{m_1=0}^{\infty}\cdots\sum_{m_{j-1}=0}^{\infty}\sum_{m_{j+1}=0}^{\infty}\cdots\sum_{m_n=0}^{\infty} Z_1{}^{m_1}\ldots Z_{j-1}{}^{m_{j-1}} Z_{j+1}{}^{m_{j+1}}\ldots Z_n{}^{m_n}$$

$$A(Z_j) = \mu_j \sum_{l=0}^{B-2}\sum_{k=2(l+1)}^{B+1} k\, P_0\,(\text{m-}(m_j - \text{k})\, e_j)\, Z_j{}^l - \lambda_j \sum_{k=1}^{B-1} C_{jk}\, P_0\,(\text{m-}\, m_j e_j)$$

From (6), we find that

$$\emptyset_j(Z, x) = \emptyset_j(Z, 0)\,[\,1\text{-}\, B_j(x)]\, e^{-sx} \quad (8)$$

where $\sum_{i=1}^{n}\lambda_i\,(1\text{-}\,C_i(Z_i))$ is denoted by s.

From (8),it follows that

$$\emptyset_j(Z) = \int_0^\infty \emptyset_j(Z, x)\, dx = \emptyset_j(Z, 0)\,\frac{1-\beta_j(s)}{s} \quad (9)$$

With the help of equations (8) and (9), equations (5) and (7) can be rewritten as follows:

$$\lambda\emptyset_0(Z) + \sum_{i=1}^{n}\mu_i Z_i \frac{\partial\emptyset_0(Z)}{\partial Z_i} = \sum_{i=1}^{n}\frac{s\beta_i(s)}{1-\beta_i(s)}\emptyset_i(Z)$$

(10)

$$\frac{\lambda_j}{Z_j{}^B}\,[\,C_j(Z_j) - \sum_{k=1}^{B-1} Z_j{}^k C_{jk}\,]\,\emptyset_0(Z) + \mu_j \sum_{k=0}^{B-1}\frac{1}{Z_j{}^k}\frac{\partial\emptyset_0(Z)}{\partial Z_j} - □\,(\text{A}(Z_j)) = \frac{s}{1-\beta_j(s)}\emptyset_j(Z)$$

(11)

Multiplying (10) by $Z_i{}^{k-1}$ and summing it over k=1 to k=B, we get

$$\sum_{k=1}^{B}\sum_{i=1}^{n}\lambda_i Z_i{}^{k-1}\emptyset_0(Z) + \sum_{k=1}^{B}\sum_{i=1}^{n}\mu_i Z_i{}^k \frac{\partial\emptyset_0(Z)}{\partial Z_i} = \sum_{k=1}^{B}\sum_{i=1}^{n}\frac{s\beta_i(s)}{1-\beta_i(s)} Z_i{}^k\emptyset_i(Z)$$

(12)

Again, multiplying (11) by $Z_i{}^B$ and then summing over j = 1,2,…,n, we have

$$\sum_{j=1}^{n}[\lambda_j C_j(Z_j) - \lambda_j \sum_{k=1}^{B-1} Z_j{}^k C_{jk}\emptyset_0]\,\emptyset_0(Z) + \sum_{k=1}^{B}\sum_{j=1}^{n}\mu_j Z_j{}^k \frac{\partial\emptyset_0(Z)}{\partial Z_j} - \sum_{j=1}^{n}\Pi(\text{A}(Z_j))\, Z_j{}^B$$

$$= \sum_{j=1}^{n}\frac{s\, Z_j{}^B}{1-\beta_j(s)}\emptyset_j(Z)$$

(13)

In order to find the channel state distribution, we subtract (13) from (12), and via some tedious algebra, we get

$$\emptyset_0(Z) = \frac{\sum_{j=1}^{n} \emptyset_j(Z) \frac{s}{1-\beta_j(s)} [\sum_{k=1}^{B} Z_j^{\,k} \beta_j(s) - Z_j^{\,B}] - \sum_{j=1}^{n} \Pi(A(Z_j)) Z_j^{\,B}}{s + \sum_{j=1}^{n} \sum_{k=1}^{B-1} \lambda_j Z_j^{\,k} (1+C_{jk})} \quad (14)$$

On setting Z=e, (14) yield

$$\emptyset_0(e) \sum_{j=1}^{n} \sum_{k=1}^{B-1} \lambda_j (1 + C_{jk}) = (B-1) \sum_{j=1}^{n} \frac{\emptyset_j(e)}{\beta_{j1}} \sum_{j=1}^{n} L_j \quad (15)$$

where

$$L_j = \square [\mu_j \sum_{l=0}^{B-2} \sum_{k=2(l+1)}^{B+1} k P_0(m - (m_j - k)e_j) - \lambda_j \sum_{k=1}^{B-1} C_{jk} P_0(m - m_j e_j)]$$

Setting Z=e in (11) and after summing it over j= 1,2,…,n, adding with (15), we get

$$\sum_{j=1}^{n} \mu_j \frac{\partial \emptyset_0(e)}{\partial Z_j} = \sum_{j=1}^{n} \frac{\emptyset_j(e)}{\beta_{j1}} - \square \emptyset_0(e) \quad (16)$$

Again, summing up (11) over j = 1,2,…n and subtracting from (10), we have

$$\sum_{j=1}^{n} [\lambda_j C_j(Z_j) - \frac{\lambda_j}{Z_j^{\,B}} C_j(Z_j) + \sum_{k=1}^{B-1} Z_j^{\,k} C_{jk}] \emptyset_0(Z) + \sum_{j=1}^{n} \mu_j [Z_j - \sum_{k=0}^{B-1} \frac{1}{Z_j^{\,k}}] \frac{\partial \emptyset_0(Z)}{\partial Z_j} + \sum_{j=1}^{n} \Pi(A(Z_j))$$

$$= \sum_{j=1}^{n} \lambda_j (C_j(Z_j) - 1) N(Z) \quad (17)$$

where $N(Z) = \sum_{i=0}^{n} \emptyset_i(Z)$

Now differentiating (11) with respect to Z_i at the point Z = e,

$$\frac{\partial \emptyset_j(e)}{\partial Z_i} - \frac{1}{2} \lambda_j \overline{C_j} \beta_{j2} \emptyset_j(e) = B \quad \mu_j \beta_{j1} \frac{\partial^2 \emptyset_0(e)}{\partial Z_i \partial Z_j} + \beta_{j1} [\lambda_j (1 - \sum_{k=1}^{B-1} C_{jk}) - \delta_{ij} B \mu_j] \frac{\partial \emptyset_0(e)}{\partial Z_j} + \lambda_j \beta_{j1} \delta_{ij} (\overline{C_j} - B) \emptyset_0(e) - \beta_{j1} \lambda_j \sum_{k=1}^{B-1} (k-1) C_{jk} - \beta_{j1} \frac{\partial^2}{\partial Z_i \partial Z_j} \square(A(Z_j))_{/Z=e} \quad (18)$$

Again, on differentiation of (17) with respect to Z_iZ_j at the point Z=e, and using the normalization condition $\sum_{i=1}^{n} \emptyset_i(e) = 1$, we have

$$[\sum_{j=1}^{n} \sum_{k=1}^{B-1} C_{jk} - \delta_{ij} \mu_i \frac{(B+2)^2 - 3B}{2}] \frac{\partial^2 \emptyset_0(e)}{\partial Z_j \partial Z_i}$$

$$= \lambda_i \overline{C_i} N_j + N_j \overline{C_j} N_i \delta_{ij} + 2\delta_{ij} \overline{C_i} (B+1) - (1+\delta_{ij})(B \lambda_i + \sum_{k=1}^{B-1} (B+1+k) C_{ik}) \frac{\partial \emptyset_0(e)}{\partial Z_j}$$

$$- \delta_{ij} \mu_i ((B+1)(B+2) - \sum_{k=0}^{B-1} (B-k)(B-k+1) \frac{\partial \emptyset_0(e)}{\partial Z_j} + \delta_{ij} (B\lambda_i (B+1+2\overline{C_i})$$

$$+ \sum_{k=1}^{B-1} (k+B)(k+B+1) C_{ik} \emptyset_0(e) - \sum_{j=1}^{n} \frac{\partial^2}{\partial Z_j \partial Z_j} \square(A(Z_j))_{/Z=e} - (B+1)(1+\delta_{ij}) \frac{\partial}{\partial Z_j} \square(A(Z_i))_{/Z=e}$$

$$- B(B+1) \square(A(Z_i))_{/Z=e} \quad (19)$$

After summing (18) over j=1,2,…,n and using the relations (16) and (19), N_i's can be determined.

CONCLUSION

Mean queue length of customers are found for each class of arrivals for the proposed model of batch arrival,batch service retrial queue which is a very important aspect of a queuing system and provides an insight how the queue is behaving. Those, who are responsible for the management of queuing

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infrastructure may get the clue for optimum utilization of resources either human or material. It also indicates the requirement of extra servers or under-utilization of service facility. A multiclass batch arrival of random size is considered, which allows inclusion of a wide variety of cases for example a fixed size of batch.

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